

On the Proton charge extensions

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Abstract

It is shown that the recent determination of the various proton charge extensions is compatible with Standard Model expectations.

A proton charge extension is defined according to its impact on some specific measurement. The impact of the parameter r_p in

$$F_1^p(\vec{q}^2) = 1 - \frac{\vec{q}^2}{6} r_p^2 \quad (1)$$

on the energy level E_n of the hydrogen atom is commonly given by [1]

$$\Delta E_n = \frac{2}{3} \pi \alpha |\psi_n(0)|^2 r_p^2 \quad (2)$$

It turns out, however, that $r_{p,\mu} < r_{p,e}$, which is considered [2] as a ‘Proton Radius Puzzle’.

Considering the different extensions of the muon and the electron wave functions in the corresponding hydrogen atoms, it is clear that the parametrization of $F_1^p(\vec{q}^2)$ should depend on two free parameters, e.g.

$$F_1^p(\vec{q}^2) = 1 - \frac{\vec{q}^2}{6} r_p^2 + \frac{(\vec{q}^2)^2}{6} \tilde{r}_p^4, \quad (3)$$

implying that

$$\Delta E_{n,\ell} = \frac{2}{3} \pi \alpha |\psi_{n,\ell}(0)|^2 r_{p,\ell}^2 \quad (4)$$

where $\ell = e, \mu$ and where

$$r_{p,\ell}^2 = r_p^2 - \langle \vec{q}^2 \rangle_{n,\ell} \tilde{r}_p^4, \quad (5)$$

with

$$\langle \vec{q}^2 \rangle_{n,\ell} = |\psi_{n,\ell}(0)|^{-2} \int \frac{d^3 q}{(2\pi)^3} \vec{q}^2 \int d^3 r e^{i \vec{q} \cdot \vec{r}} |\psi_{n,\ell}(\vec{r})|^2. \quad (6)$$

Thus $\langle \vec{q}^2 \rangle_{n,\mu} > \langle \vec{q}^2 \rangle_{n,e}$, accounts for the observed [2] $r_{p,\mu} < r_{p,e}$, in full agreement with Standard Model expectations.

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References

1. R. Karplus, A. Klein, and J. Schwinger, Phys. Rev. **86** (1952) 1183.
2. R. Pohl et al., Ann. Rev. Nucl. Part. Sci. **63** (2013) 175, and references therein.